Three-color graph of the Morse flow on a compact surface with boundary

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We consider the Morse flows [1] (Morse-Smale flows without closed orbits) on the compact surfaces with boundary. There was constructed a complete topological invariant of these flows – an equipped three-colored graph.

The graph T will be called *three-color graph*, if all its vertices have a degree not bigger 3, and edges are painted in three colors, so that edges of different colors converge at each vertex. Colors are denoted by the letters s, t, u.[2, 3] The vertices of three-colored graph correspond to the standard areas on the surface, that look like a curvilinear triangle or quadrilateral. There were found conditions in which a three-colored graph generates a flow.

Теорема 1. *For a connected tricolor graph having properties*

1) each edge of the graph is marked with one of the three letters: s, t, u, and each vertex is white or black;

2) two edges of the same type can not come out from each vertex;

3) for each black inner vertex there is a su -cycle of length 4 that contains it;

4) if two black vertices are connected by a $u-$ or $s-$ edge and one of them is bounded, then the other will be bound;

5) each white vertex is internal. And if it is connected to the black vertex $u-$ edge ($s-$ edge), then this black vertex will be the limit.

there exists a Morse flow on a connected surface with a boundary, the three-color graph of which is a given graph.[1]

The number of topologically non-equivalent flows with 2, 3, 4, and 5 standard areas was calculated. For each of them, the surface on which this flow is set is determined. The distribution of the number of flows on the surfaces is shown in the table.

	2 stand.areas	3 stand.areas	4 stand.areas	5 stand.areas
Disk (D^2)	5	3	18	22
Myobius leaf		1	5	15
Myobius with a hole			2	2
Ring($S^1 \times I$)	1		10	10
Ring with a hole			1	1
Klein bottle				3
Torus with a hole				1

Література

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